# Effective Thermal Conductivity of Sintered Metal Fibers

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**For the effective thermal conductivity of dry and fluid-saturated sintered metal fibers, existing correlations are compared to experimental data, and a new empirical expression that compensates for the effect of the aspect ratio of the fibers is presented. It is shown that the present equation predicts the data with 10% deviation, while the Acton equation, which has been recommended in some references, has approximately 200% error.**

## Nomenclature

$A$ = constant in Eq. (7)
$a$ = constant in Eq. (11)
$b$ = constant in Eq. (11)
$c$ = $0.5 + A\cos(\varphi/3)$
$d$ = fiber diameter, $\mu$
$k_e$ = effective thermal conductivity, W/m-K
$k_f$ = fluid thermal conductivity, W/m-K
$k_s$ = solid thermal conductivity, W/m-K
$l$ = fiber length, $\mu$
$m$ = empirical value in Eq. (14)
$n$ = empirical value in Eq. (3)
$\alpha$ = constant in Eq. (10)
$\epsilon$ = porosity
$\nu$ = $k_f/k_s$, conductivity ratio
$\varphi$ = value in Eq. (7)

## Introduction

POROUS media have various applications in many technical areas, including transpiration cooling, heat pipes, electronic cooling, heat exchangers, food processing, refrigeration and air conditioning, and aerospace. Because of high strength, high effective thermal conductivity, and low capillary radius, sintered metal fibers are the best choice in some of these fields. In particular, metal fibers have been recently used as a capillary structure in heat pipes and two-phase transport loops for thermal management in space. In addition, metal fibers can be formed into any desirable shape by sintering. The subject of this paper, the effective thermal conductivity of dry and fluid-saturated sintered metal fibers, has been the theme of numerous studies in the past thirty years.

Because it is not a transport property, the effective thermal conductivity of any heterogeneous material is normally defined as the heat flux divided by the temperature drop per unit length. This study is concerned with the prediction of effective conductivity in the direction "across the fibers," i.e., perpendicular to the felting plane, as a function of solid conductivity $k_s$, fluid conductivity $k_f$, porosity $\epsilon$, and the aspect ratio $d/l$. Here, $d$ and $l$ indicate the diameter and length of the fibers, respectively. Notice that "along the fibers" in the literature refers to the direction parallel to the felting plane. The equation presented herein is for dry and fluid-saturated metal fibers, where dry refers to those in a vacuum.

An effectual model on the thermal conductivity has eluded researchers mainly for three reasons: random distribution of fibers, the effect of aspect ratio, and the effect of conductivity ratio $k_f/k_s$. Unlike packed spheres or cloth, metal fibers are randomly distributed. Many researchers[1-3] have thus employed an "average" unit cell because there is no repeating unit cell to base a model on. Furthermore, some existing equations were derived including $d$ and $l$ (e.g., Ref. 1), but these parameters dropped out when solving in terms of porosity. Semena and Zaripov[3] showed that this type of simplification is inadequate. They found that the effective thermal conductivity increases with a decreasing aspect ratio. The effect of the aspect ratio must therefore be empirically compensated for.

Many models, reviewed here, have been developed under the assumptions that the direction of the net heat flow is perpendicular to the felting plane and that the heat flow parallel to the felting plane is zero. These assumptions make a one-dimensional solution possible. However, the only case where the heat flow parallel to the felting plane is absent is when $k_f = k_s$, which has a trivial solution. Unequal component conductivities ($k_f \neq k_s$) give rise to a finite heat flow along the fibers.

## Existing Correlations

The two simplest models are the parallel and series conduction models. These represent the limits of conductivity for heterogeneous materials. The upper limit, given by the parallel model, is

$$k_e = \epsilon k_f + (1-\epsilon)k_s \tag{1}$$

and the lower limit, given by the series model, is

$$k_e = \frac{1}{\epsilon/k_f + (1-\epsilon)/k_s} \tag{2}$$

Aivazov and Domashnev[4] have formulated a general equation to calculate the effective thermal conductivity of porous media in a vacuum. Their expression, based on Odelevskii's work, is

$$k_e = k_s \frac{1-\epsilon}{1+n\epsilon} \tag{3}$$

where $n$ is to be determined empirically. Aivazov and Domashnev showed that for hot-pressed titanium-nitride, $n = 6\epsilon$ provides a good approximation for thermal conductivity. Later, Koh and Fortini[5] applied the same equation to experimental data; they recommend $n = 11\epsilon$ for felt metals.

Semena and Zaripov[3] give an empirical equation for sintered metal fibers in a vacuum for a range of $0.65 < \epsilon < 1$:

$$k_e = k_s(1-\epsilon)^2 e^{-[\pi + \log_{10}(d/l)]} \tag{4}$$

It is noted from this equation that $k_e$ increases as $d/l$ decreases.

Using a simplified unit cell and square fiber cross sections, Acton[1] derived the equation

$$k_e = \epsilon^2 k_f + (1-\epsilon)^2 k_s + \frac{4\epsilon(1-\epsilon)k_s k_f}{k_s + k_f} \tag{5}$$

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In one of the earlier papers on this subject, Dul'nev and Muratova[6] derived an equation that includes radiation effect. Neglecting the radiation term in their result reduces to

$$k_e = k_s\left[c^2 + \nu(1-c)^2 + \frac{2\nu c(1-c)}{\nu c + (1-c)}\right] \tag{6}$$

where $\nu = k_f/k_s$, and $c$ is given by

$$c = 0.5 + A\cos(\varphi/3) \tag{7}$$

Here, $A$ and $\varphi$ are

$$A = -1, \qquad \varphi = \cos^{-1}(1-2\epsilon) \qquad \text{for } 0 \le \epsilon \le 0.5 \tag{8}$$

$$A = 1, \qquad \varphi = \cos^{-1}(2\epsilon - 1) \qquad \text{for } 0.5 \le \epsilon \le 1.0 \tag{9}$$

where $\varphi$ ranges from $3\pi/2$ to $2\pi$.

Alexander[7] suggests an empirical equation for porous media:

$$k_e = k_f\left(\frac{k_s}{k_f}\right)^{(1-\epsilon)^\alpha} \tag{10}$$

where $0 \le \alpha \le 1$. For air- and water-saturated fibers, he obtained $\alpha = 0.34$.

Soliman et al.[2] measured effective thermal conductivities of copper and nickel fibers in a vacuum and in water. Their model contains two empirical constants depending on fiber material:

$$k_e = k_s\left\{a(1-\nu)(1-\epsilon) + \nu\left[1 + \frac{(1-\nu)(1-\epsilon)(1-a)}{1-(1-\nu)(1-\epsilon)^b}\right]\right\} \tag{11}$$

where $a = 0.13$ and $b = 0.006$ for copper, and $a = 0.046$ and $b = 0.012$ for nickel. The model fits the data well, but empirical values are given for nickel and copper only, and the values of $b$ are validated for water-saturated fibers only.

Koh et al.[8] presented an equation for dry felt metals based on the contact resistance and the matrix network resistance in series:

$$k_e = k_s\frac{(1-\epsilon)^2}{(1+2\epsilon)(1-\epsilon)+1.5\epsilon} \tag{12}$$

Rayleigh[9] derived an equation for parallel circular cylinders. Because it is similar to metal-fiber models, it is included here:

$$k_e = k_f\frac{k_f + k_s - (1-\epsilon)(k_f - k_s)}{k_f + k_s + (1-\epsilon)(k_f - k_s)} \tag{13}$$

Most existing works, including the simple parallel and series conduction models, have the effective thermal conductivity as a function of porosity and constituent conductivities only.[1,5,7-9] Furthermore, many publications containing experimental data fail to provide other useful information besides $k_s$, $k_f$, and $\epsilon$. Unfortunately, there are additional parameters that affect the conductivity, such as length and diameter of the fibers, degree of sintering, and direction of the applied heat flow.

## Experimental Data

Table 1 gives sources of experimental data in the literature for the effective thermal conductivity of dry and fluid-saturated sintered metal fibers. Januszewski et al.[10] showed that tensile strength correlates with thermal conductivity for various sintered metals. Semena and Zaripov[3] presented the effect of aspect ratio on the conductivity and also claimed that conductivity is not affected by oxidation of the fibers. Data from Alexander[7] and Ferrell et al.,[11] for a porosity of about 0.6, show a dimensionless conductivity $k_e/k_s$ of 0.06, 0.20, and 0.26 for copper, nickel, and stainless steel, respectively. This data suggest that the dimensionless conductivity is a function of fiber material. More data, however, are required to justify such a claim.

Figure 1 shows the dimensionless conductivity for the data from the references listed in Table 1. The comparison is for cases where $k_f/k_s < 0.01$. Semena and Zaripov's data clearly follow a trend with respect to porosity and with respect to aspect ratio, except for low diameter (20-$\mu$) cases, which have higher conductivities than expected. In addition, their data compare quite well with that of Koh et al. The data from Koh and his co-workers, however, cannot be included in the following correlation since no aspect ratio is given. Alexander does not give the direction of the heat flow. Januszewski et al.[10] include neither the direction nor the aspect ratio. Soliman et al.[2] provide the aspect ratio and the direction but give a high measured error for some cases. The data of Alexander, Januszewski et al., and Soliman et al. are scattered and do not follow a trend; therefore, these are not included in the correlation that follows. Thus, the present expression is based on Semena and Zaripov's data for diameters not lower than 40 $\mu$.

It is imperative that future work be clear on terms; for instance, some works refer to a "dry" specimen as that in a vacuum,[2] while others refer to "dry" as air-saturated.[7,11] Of equal importance is the specification of the direction in which the heat flow is measured, whether parallel or perpendicular to the felting plane. All work should also include the length and diameter of the fibers.

## Analysis

Odelevskii (referred to in Ref. 4) derived the following equation:

$$\frac{k_e}{k_s} = 1 + \frac{\epsilon}{[(1-\epsilon)/m] + [k_s/(k_f - k_s)]} \tag{14}$$

where $m = 3$ for a matrix system with cubic inclusions and $m = 2$ for parallel cylinders. The present authors have correlated $m$ for metal fibers. First, $m$ was correlated by a parabolic

**Table 1 Experimental data**

| Reference | Solid | Fluid | Porosity range | Fiber size, $\mu$ | Direction of heat flow | Error, % |
|---|---|---|---|---|---|---|
| Alexander[7] <br> Ferrell et al.[11] | SS, Ni, Cu | Air <br> Water | 0.58–0.89 | $40 \le d \le 100$ <br> $1650 \le l \le 3250$ | a | 10 |
| Semena and Zaripov[3] | Cu | Vacuum <br> Water | 0.20–0.96 | $20 \le d \le 70$ <br> $3000 \le l \le 10{,}000$ | Across Fibers | 3–10 |
| Januszewski et al.[10] | SS, Ni, Hastelloy-X | Air | 0.40–0.89 | a | a | 7 |
| Soliman et al.[2] | Cu, Ni | Vacuum <br> Water | 0.66–0.90 | $16 \le d \le 43$ <br> $1600 \le l \le 4300$ | Across and along fibers | 7–20 |
| Koh et al.[8] | Ni | Vacuum | 0.44–0.77 | a | a | a |

[a]Not available.

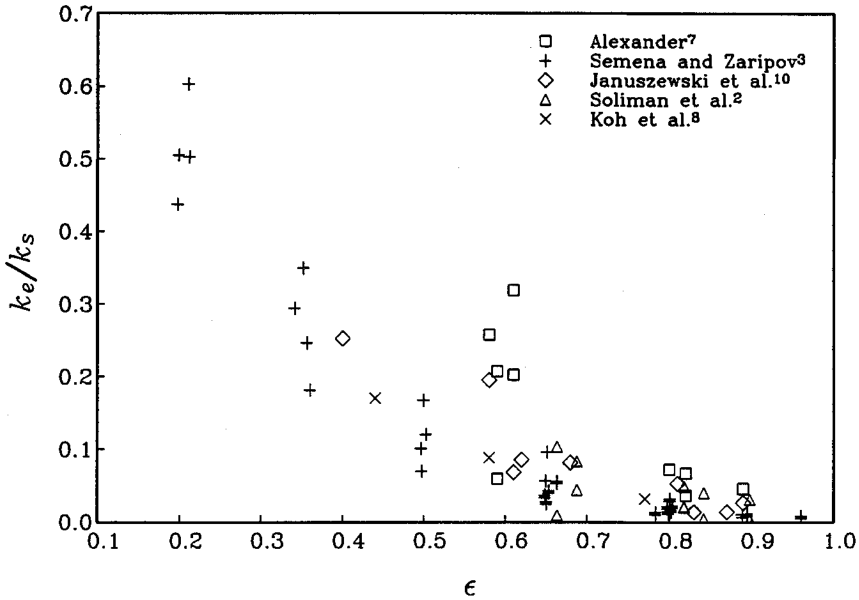


Fig. 1 Experimental dimensionless conductivity vs porosity ($k_f/k_s < 0.01$).

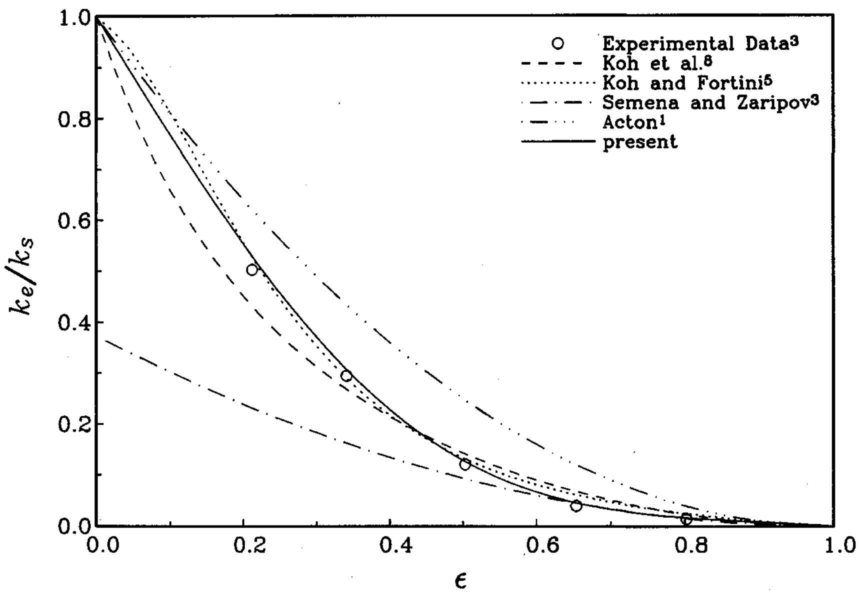


Fig. 2 Comparison of correlations for dry metal fibers ($d/l = 0.007$).

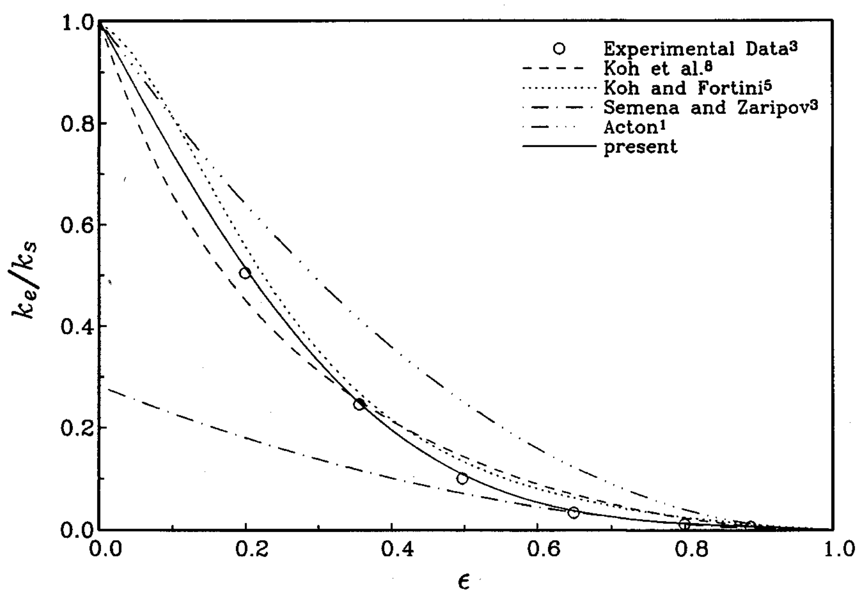


Fig. 3 Comparison of correlations for dry metal fibers ($d/l = 0.013$).

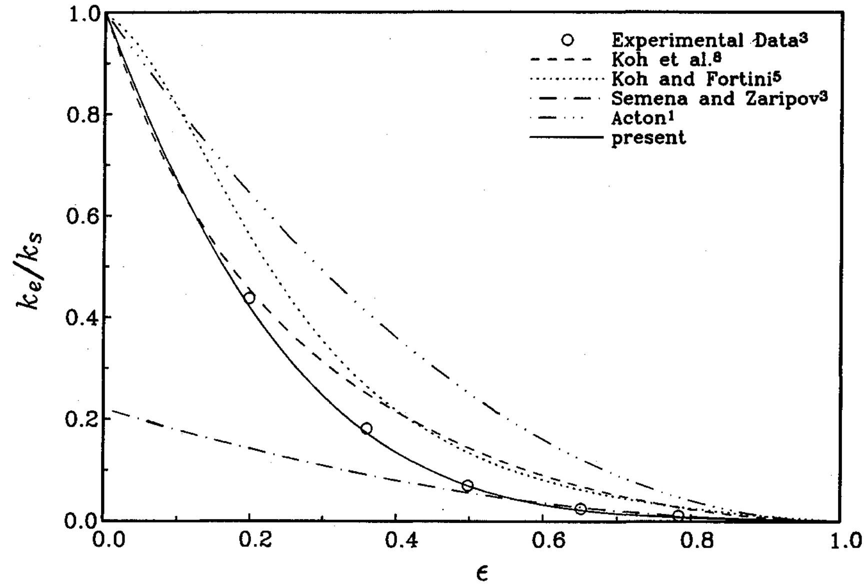


**Fig. 4 Comparison of correlations for dry metal fibers ($d/l = 0.023$).**

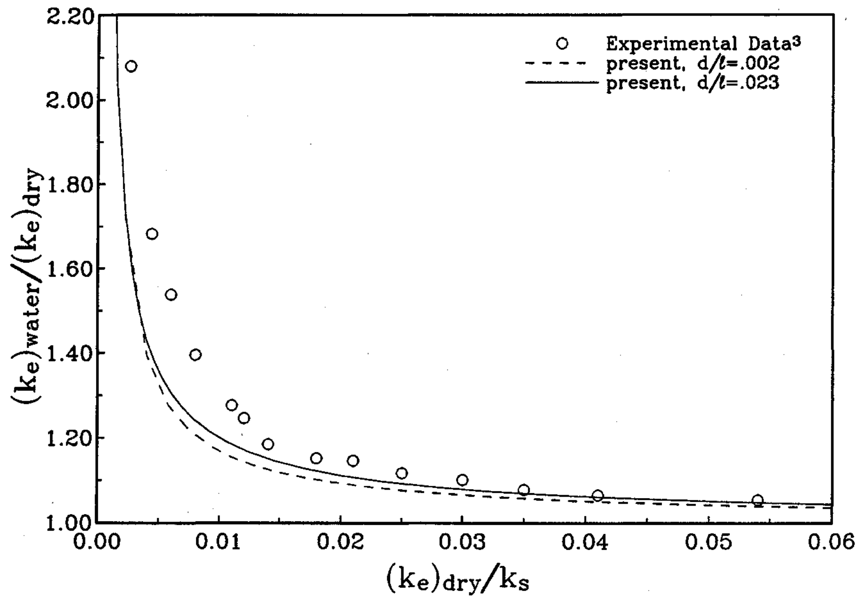


**Fig. 5 Comparison of the present correlation for copper fibers saturated with water.**

**Table 2 Standard deviation from experimental data**

| Correlations | Standard deviation, % |
|---|---|
| Acton[1] | 196.1 |
| Soliman et al.[2] | 81.2 |
| Semena and Zaripov[3] | 36.9 |
| Koh and Fortini[5] | 77.3 |
| Dul'nev and Muratova[6] | 368.5 |
| Koh et al.[8] | 79.1 |
| Present | 10.1 |

least-squares procedure with respect to $\epsilon$, resulting in three coefficients, $C_1$, $C_2$, and $C_3$, as follows:

$$m = C_1(C_2 - \epsilon)^2 + C_3 \tag{15}$$

Two of these coefficients, $C_1$ and $C_3$, were then recorrelated linearly with respect to $d/l$, and the final expression is

$$m = [1.2 - 29(d/l)](0.81 - \epsilon)^2 + [1.09 - 2.5(d/l)] \tag{16}$$

The data used in the present correlation are from Semena and Zaripov,[3] which were obtained across dry copper fibers with diameters no less than 40 $\mu$. Using Eq. (14) with Eq. (16) provides consistent results over an aspect ratio range of 0.004–0.023.

Figures 2–4 compare dimensionless conductivity ($\nu = 0$) as a function of porosity for some of the expressions provided herein. Note that some equations have a very large error in the high-porosity range. For example, at a porosity of 0.8 the equation of Koh et al.[8] predicts up to twice as high as the data, while the Acton equation predicts twice to four times as high.

Table 2 gives the standard deviation from Semena and Zaripov's experimental data for the different correlations. Those expressions that are not included in Table 2 predict zero conductivity for dry fibers, and thus these expressions will inherently yield low conductivities for any sintered metal application where $k_f/k_s \ll 1$. Note that the Acton equation, which is recommended in some recent references for heat pipe applications,[12,13] predicts the data very poorly. This is due to the large solid-to-solid contact area in his derivation.

The present equation can also be applied to fluid-saturated sintered metal fibers. Figure 5 shows the ratio of water-saturated conductivity ($k_f/k_s = 0.0015$) to dry conductivity vs the dimensionless dry conductivity. The data represent cases with an aspect ratio range of 0.002–0.023.[3] Note that the data indicate that the ratio shown in Fig. 5 is a weak function of aspect ratio.

## Conclusions

An expression for the effective thermal conductivity of metal fibers is presented based on the work of Odelevskii. More experimental data are certainly required to verify and generalize the present equation. However, it is recommended that Eqs. (14) and (16) be used to calculate the thermal conductivity of dry and fluid-saturated sintered metal fibers at the present time.

Future studies should provide measured data on the effective thermal conductivity of sintered metal fibers for $k_f/k_s > 0.01$, high porosity, and low fiber diameter (especially less than 40 $\mu$).

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